A new Model for Karst Spring Hydrograph Analysis

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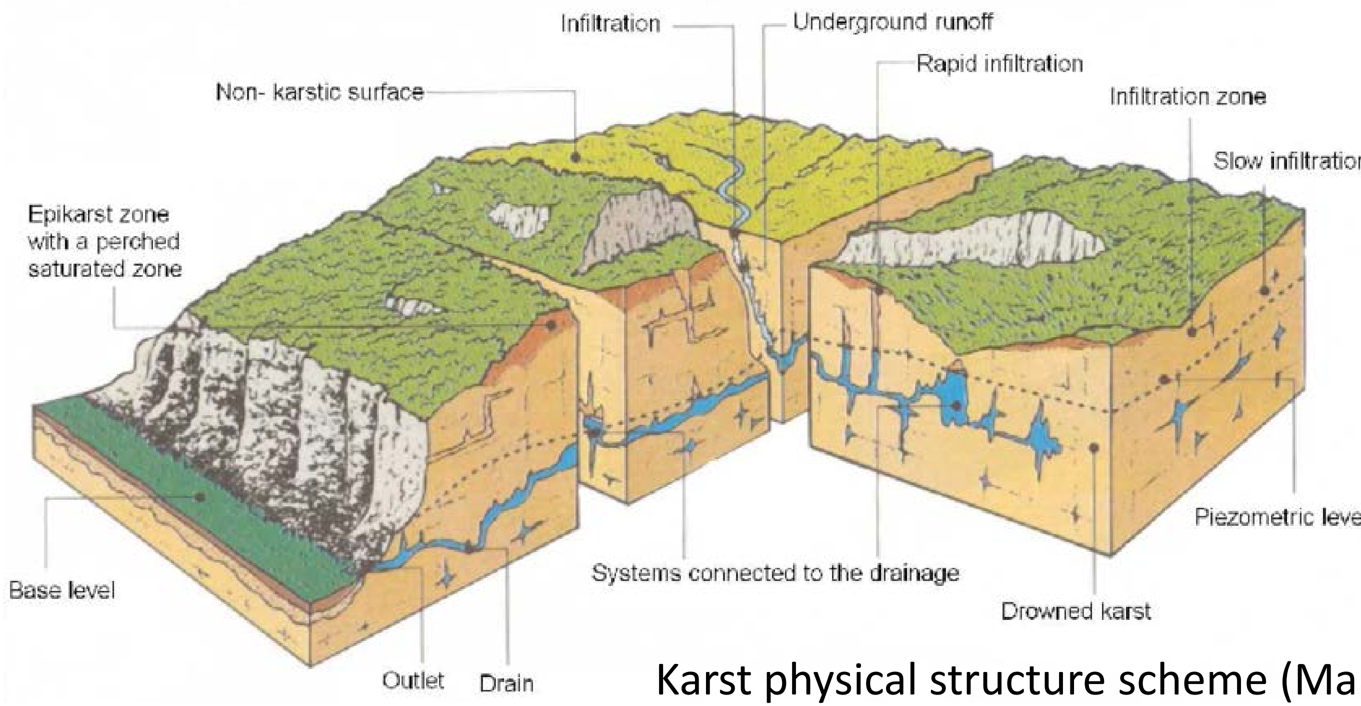
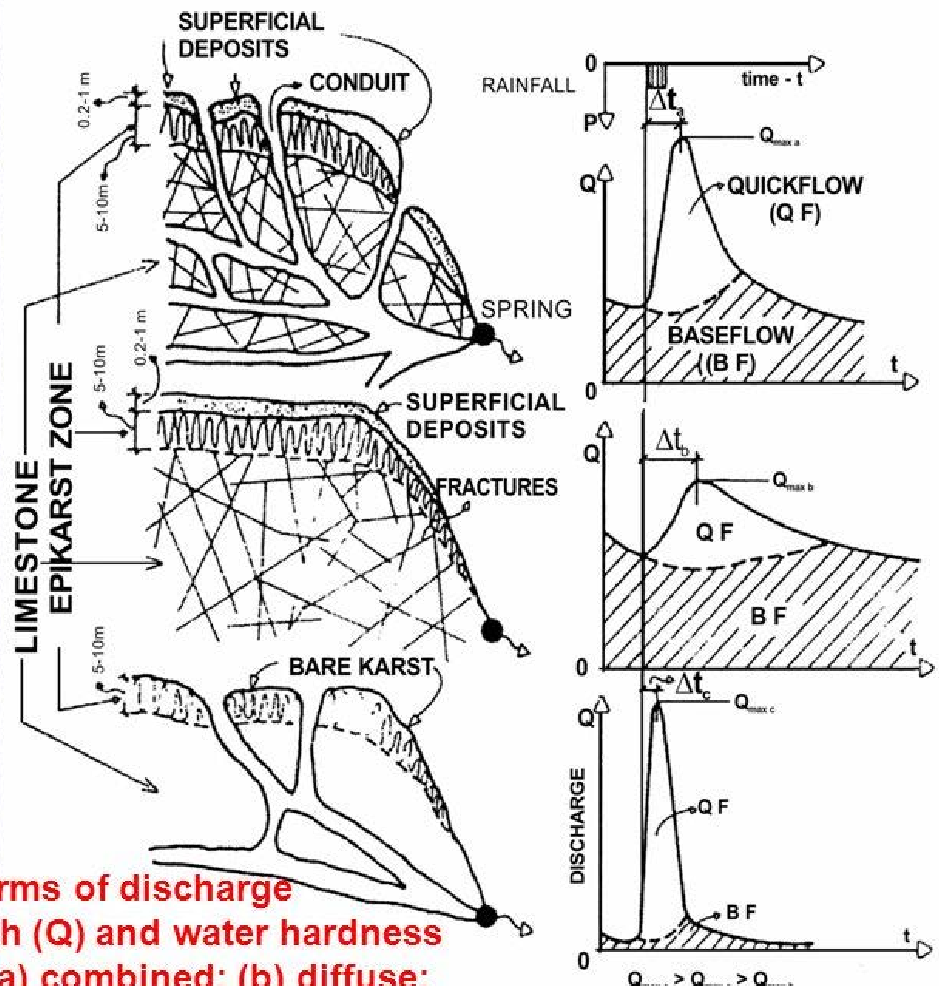
Characterization of Karst Hydrogeology



Karst physical structure scheme (Mangin, 1975)

- Karst environment is very different from other environments in terms of water flow and storage.
- Direct methods for characterization: speleology, cave diving, camera recording, borehole logging, remotely-controlled vehicles, tracer experiments, …

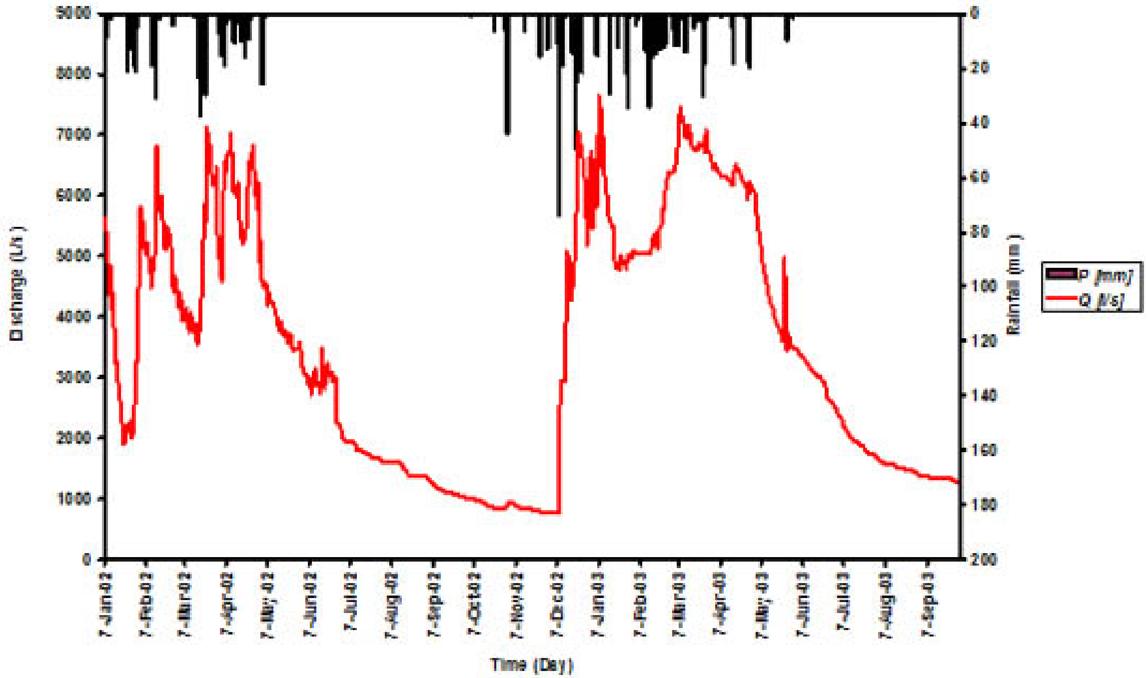
Karst Spring Hydrograph: An Indicator of Karst Aquifers



Various forms of discharge hydrograph (Q) and water hardness (WH) for; (a) combined; (b) diffuse; and (c) conduit type karst springs, as a reaction to the same rainfall (P)

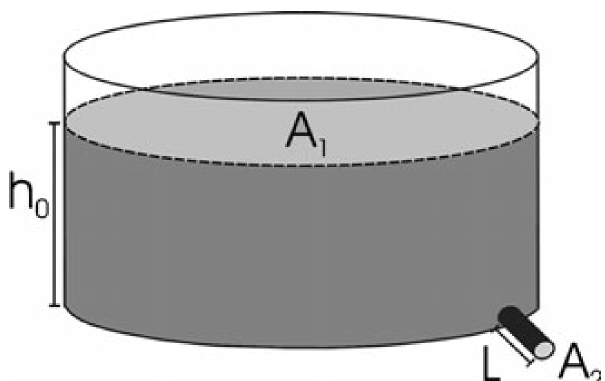
- **Natural experiments** of rainfall and karst aquifer responses occur every day.
- **karst spring hydrograph:** the discharge hydrograph appearing at a spring in a karst region where surface flow is almost not possible due to well-developed surface and underground karst landforms (Bonacci, 1993).
- Different hydrographs for different karst types of karst springs.

Hydrograph Recession Curve



Karst aquifers do not have a strong capability of storing water.

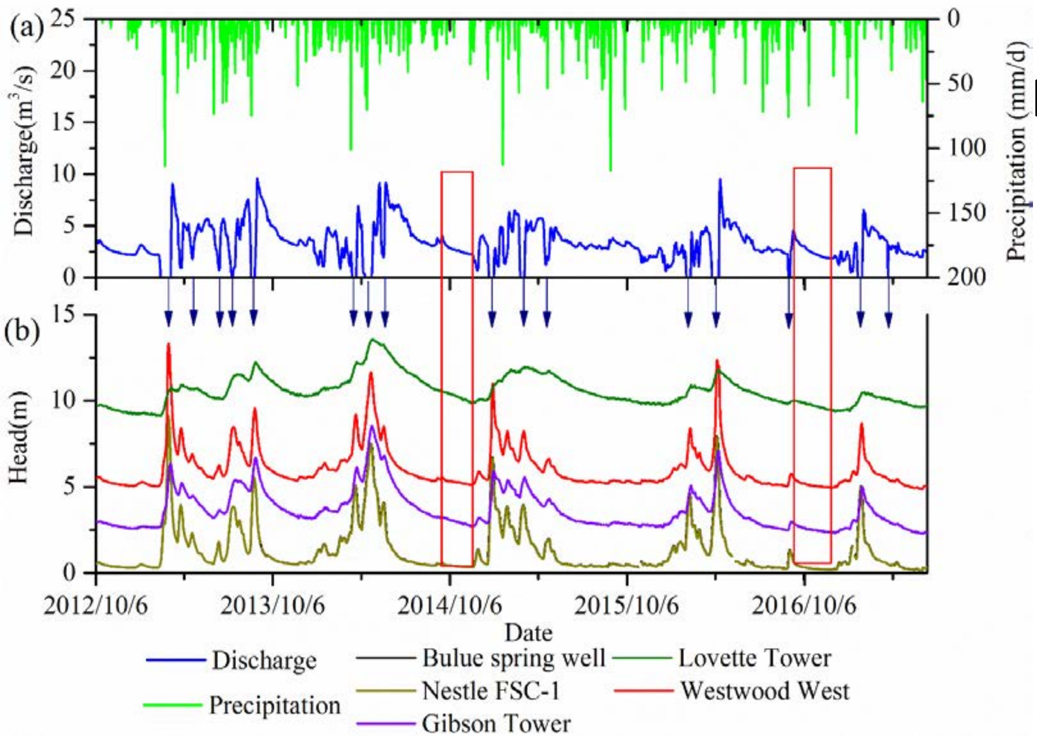
Hydrograph recession curve: in a long-lasting period with no precipitation



$$Q_t = Q_0 e^{-\alpha t}$$

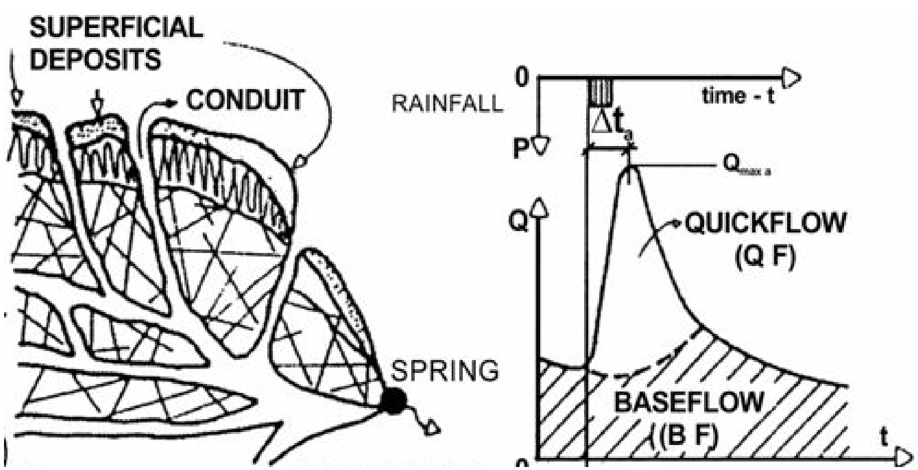
Maillet (1905)

Florida Hydrograph Recession Curve



Hydrograph of Madison Blue Spring

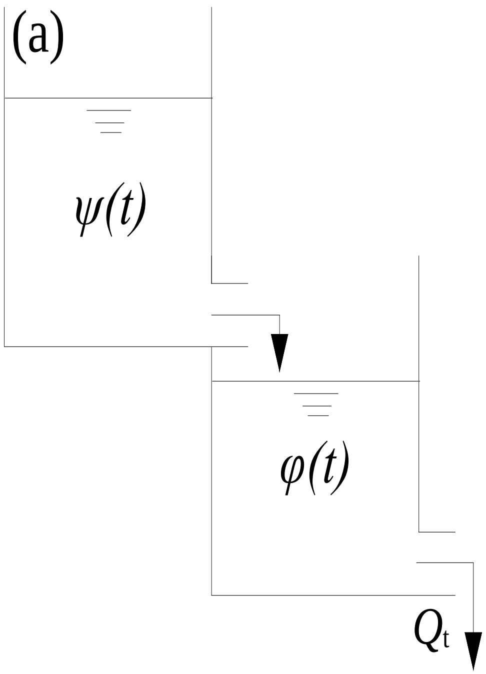
More frequent rainfall events



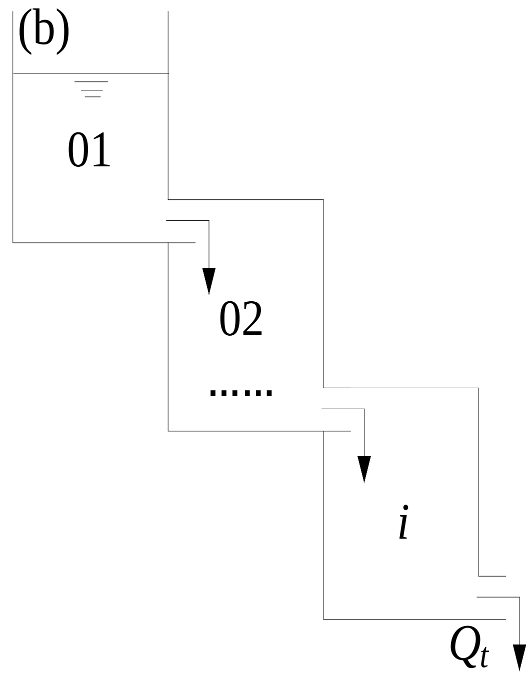
More complicated karst hydrogeology

Models of Hydrograph Recession Curve

Mangin Model (1975)



Fiorillo Model (2011)



Torricelli reservoir (conduit)

Darcy reservoir (matrix)

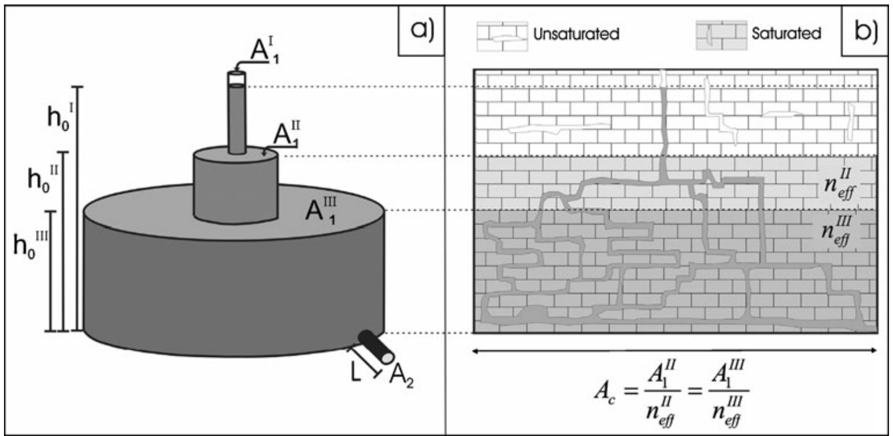
Poiseuille reservoir (fracture)

Conduit reservoir:

Unsaturated zone flow, $\Psi(t)$

Matrix reservoir:

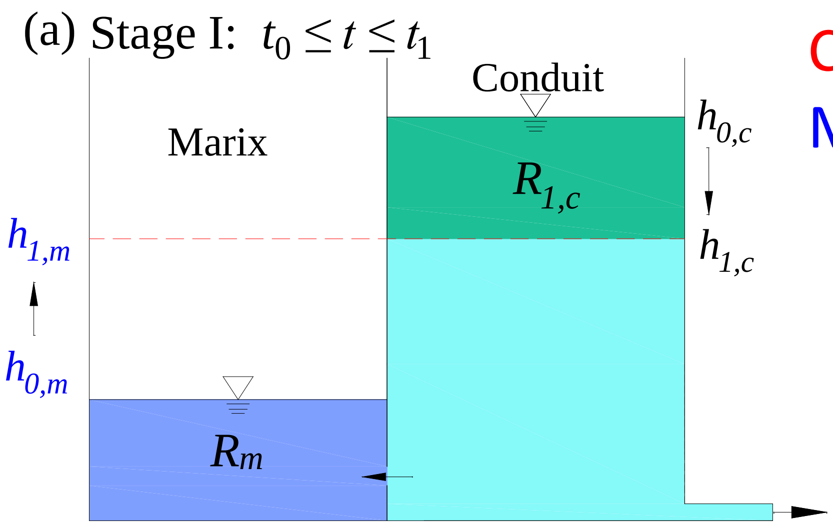
Saturated zone flow: $\varphi(t)$



Our Conceptual Model of Flow Dynamics

Two reservoirs: Conduit and matrix (including fracture)
Three stages:
Conduit-flow-dominated, mixed-flow, and matrix-flow-dominated
Explicitly separated conduit and matrix flows in the mixed-flow stage

Conduit-flow-dominated Stage

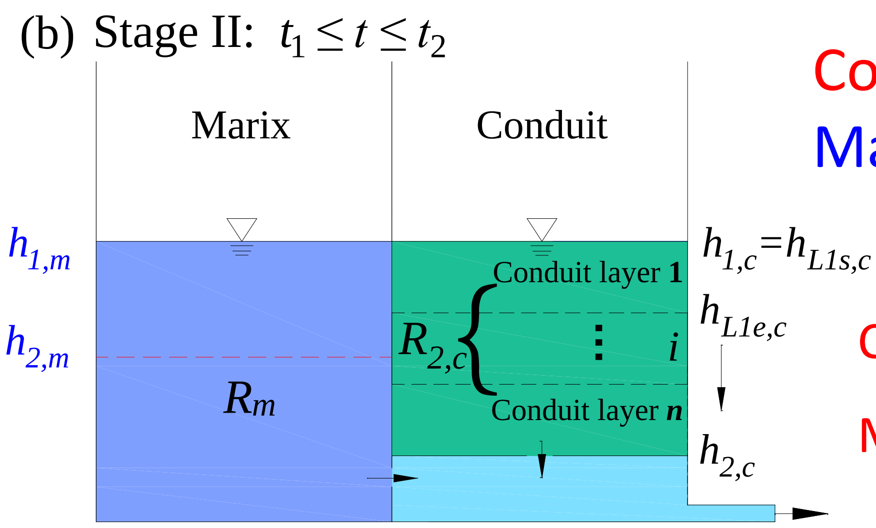


Conduit: head decreases from $h_{0,c}$ to $h_{1,c}$
Matrix: head increases from $h_{0,m}$ to $h_{1,m}$

$$h_{1,c} = h_{1,m}$$

$$Q_t^I = Q_0^I - \gamma t$$

Mixed-flow Stage with Multiple Conduit Layers



Conduit: head decreases from $h_{1,c}$ to $h_{2,c}$

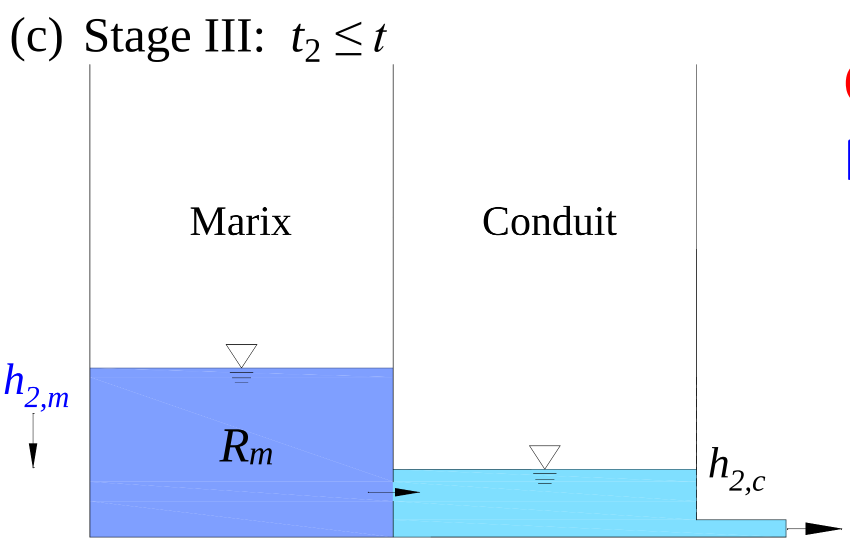
Matrix: head increases from $h_{1,m}$ to $h_{2,m}$

$$h_{2,c} \neq h_{2,m}$$

Condit flow $$Q_{t,Li,c}^{II} = Q_{Lis,c} - \beta_i (t - t_{Lis,c})$$

Matrix flow $$Q_{t,m}^{II} = Q_{1,m} e^{-\alpha_1 (t - t_1)}$$

Matrix-flow-dominated Stage



Conduit: head remains at $h_{2,c}$

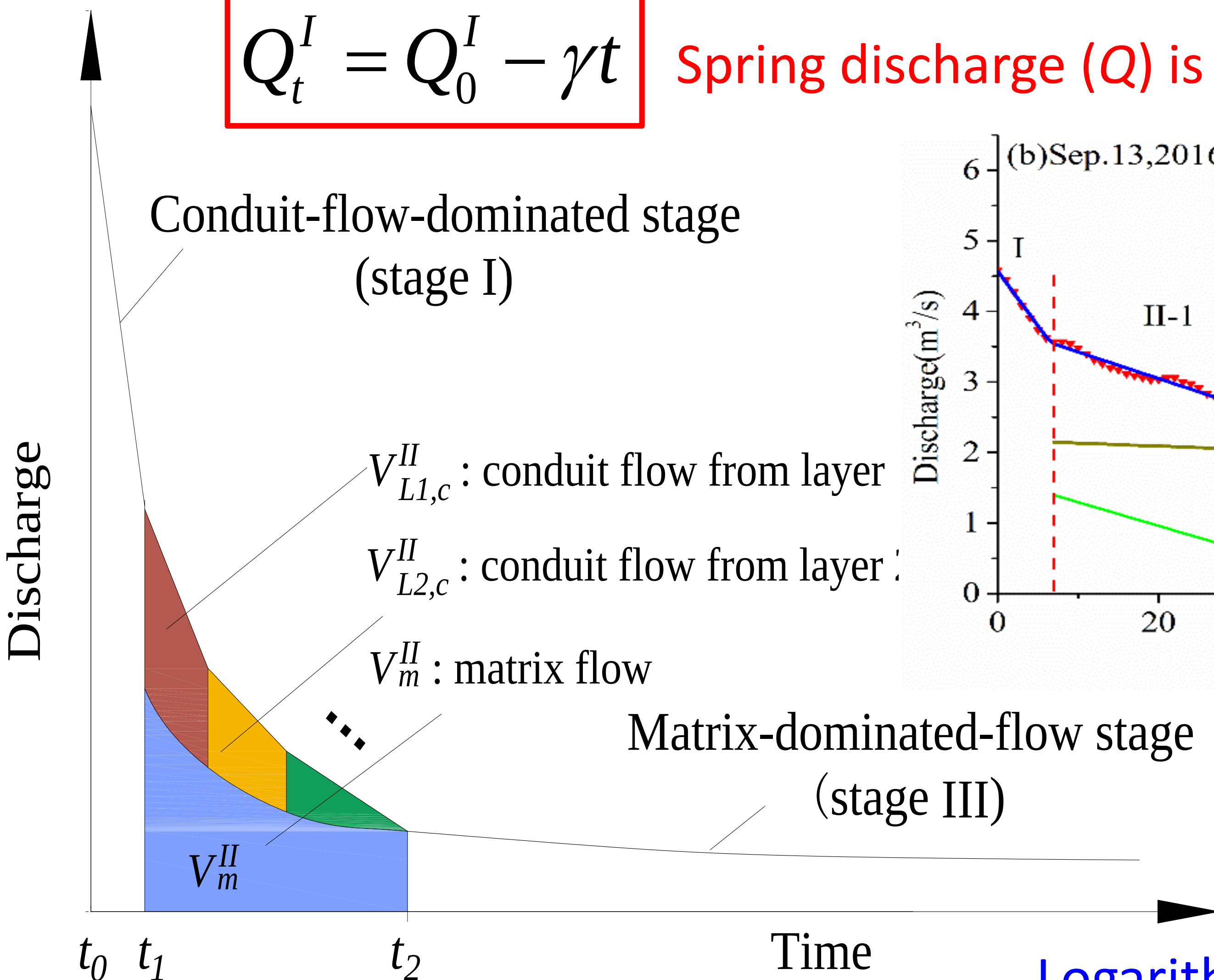
Matrix: head continues decreasing

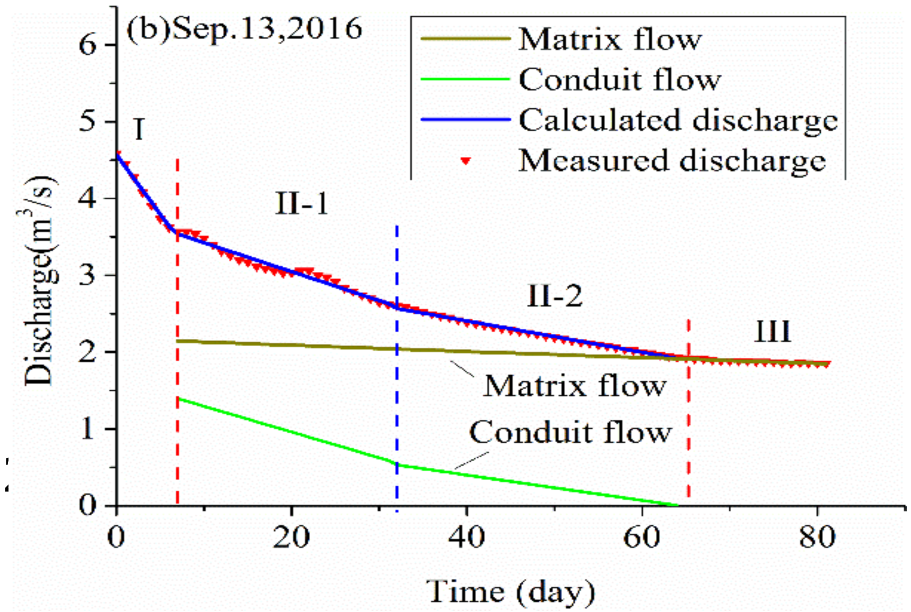
$$Q_t^{III} = Q_{2,m} e^{-\alpha_2 (t - t_2)}$$

Idealized Hydrograph Separation

$$Q_t^I = Q_0^I - \gamma t$$

Spring discharge (Q) is linear with time (t).



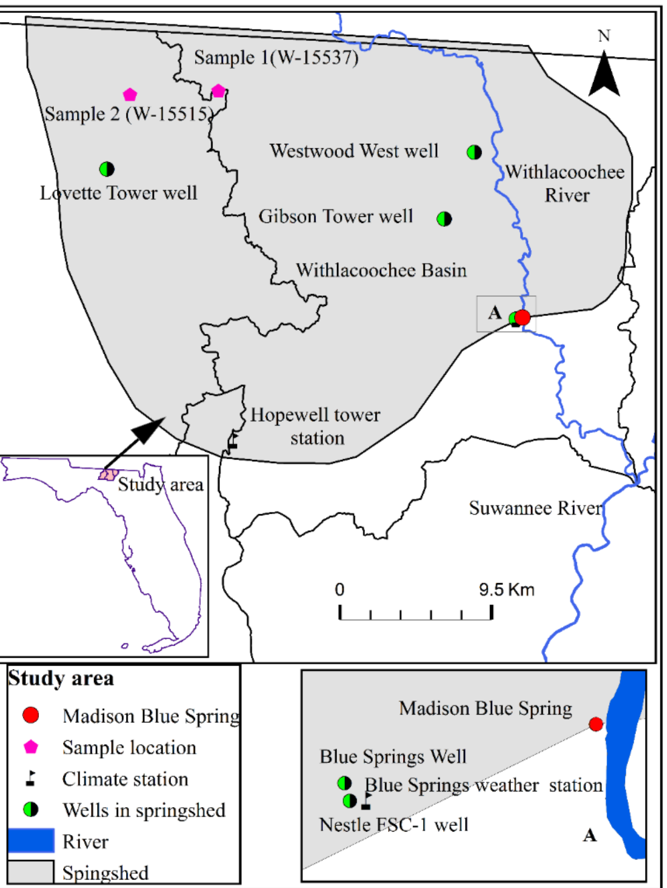


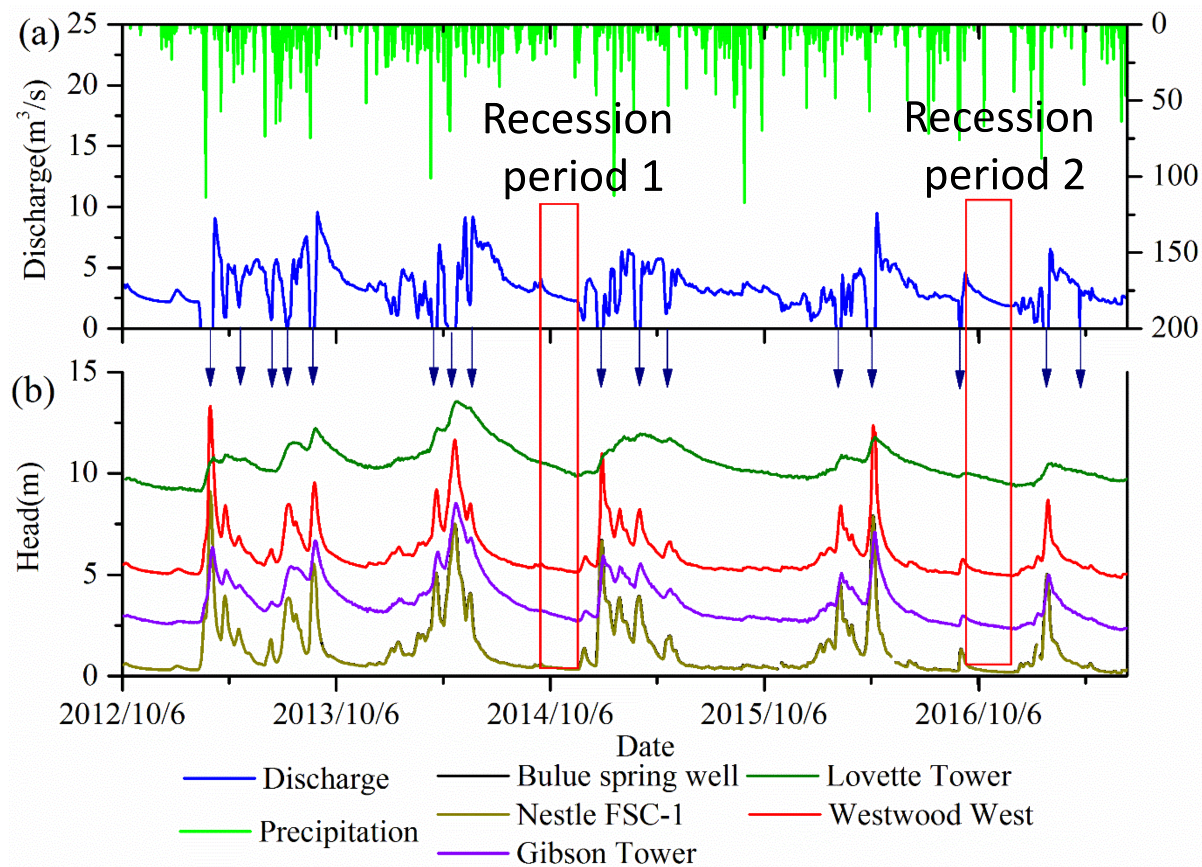
$$Q_t^{III} = Q_{2,m} e^{-\alpha_2 (t - t_2)}$$

Logarithm of spring discharge ($\log Q$) is linear with time (t).

Real-World Application and Model Comparison

- Madison Blue Spring located in SRWMD
- Two periods are selected for model application and evaluation
  - Recession period 1: small conduit flow
  - Recession period 2: large conduit flow





Recession Period 1

When conduit flow is small, the three models fit the data almost equally well.

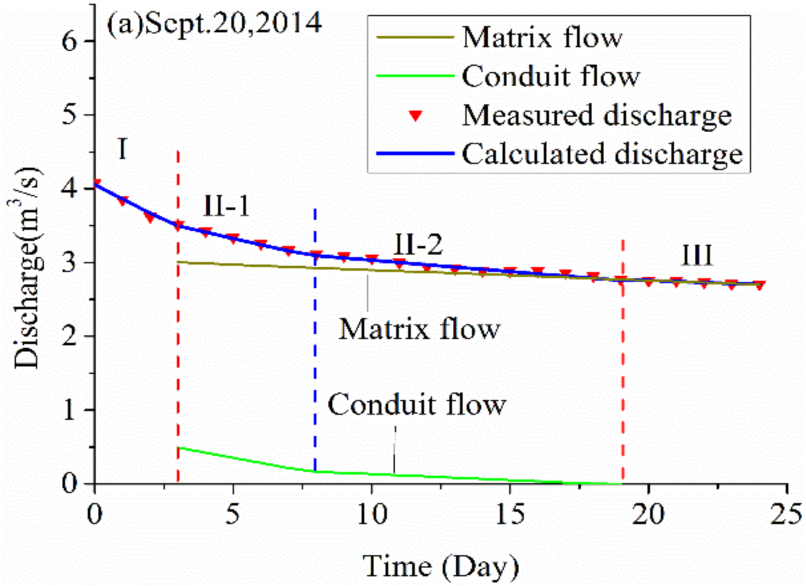
$$\text{misfit} = \sum_{i=1}^{n} |r_i|$$

Our model: 0.433 m³/s

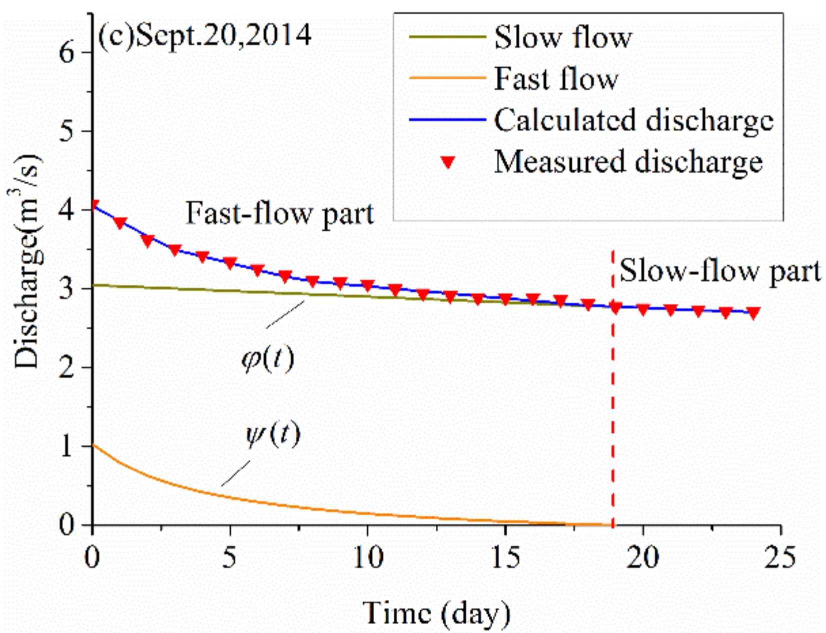
Mangin model: 0.462 m³/s

Fiorillo model: 0.449 m³/s

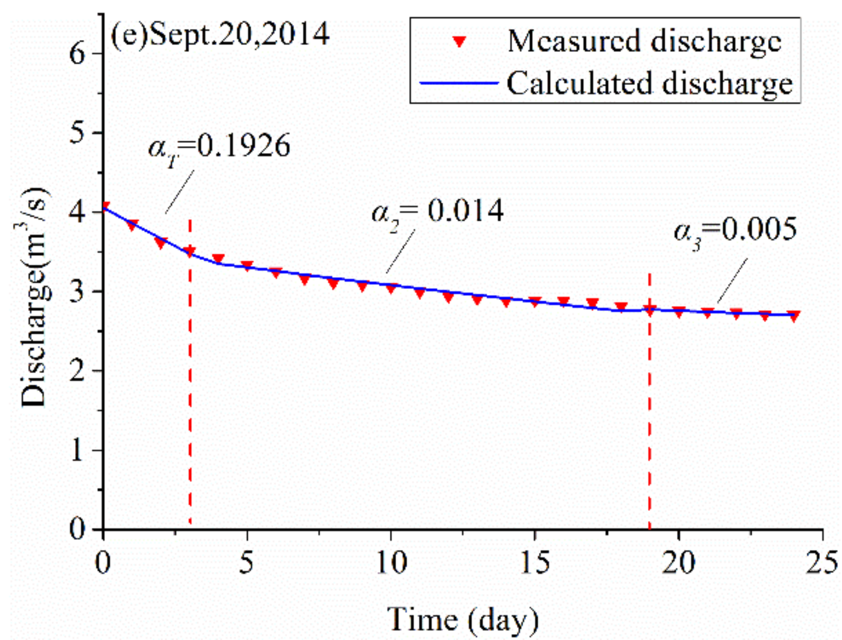
Our model



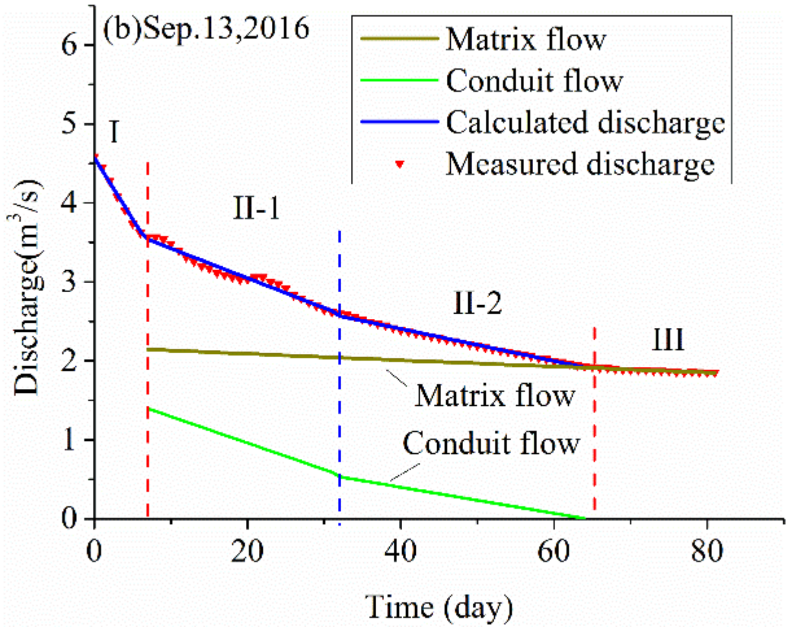
Mangin model (1975)



Fiorillo model (2011)



Our model



Recession Period 2

When conduit flow is large, our model outperforms the other two models.

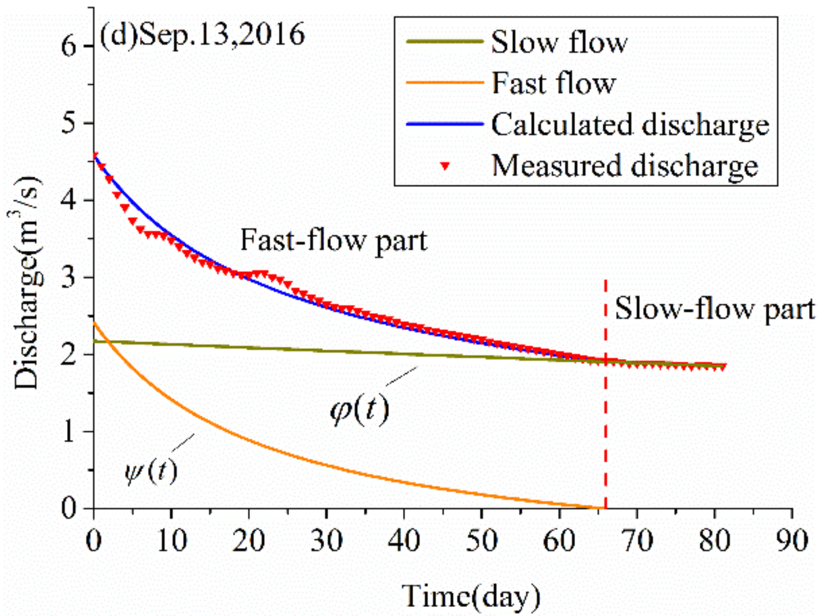
$$\text{misfit} = \sum_{i=1}^{n} |r_i|$$

Our model: 1.895 m^3/s

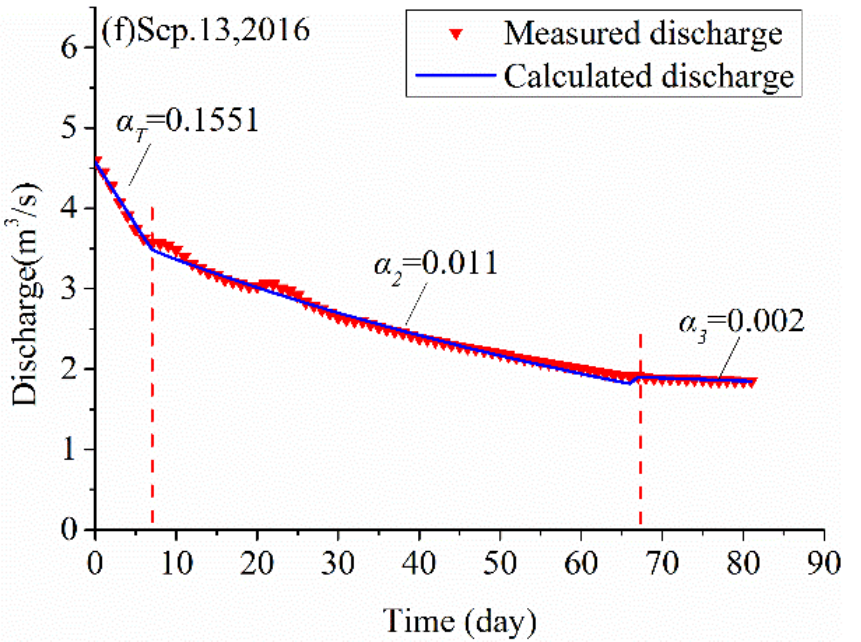
Mangin model: 4.286 m^3/s

Fiorillo model: 2.747 m^3/s

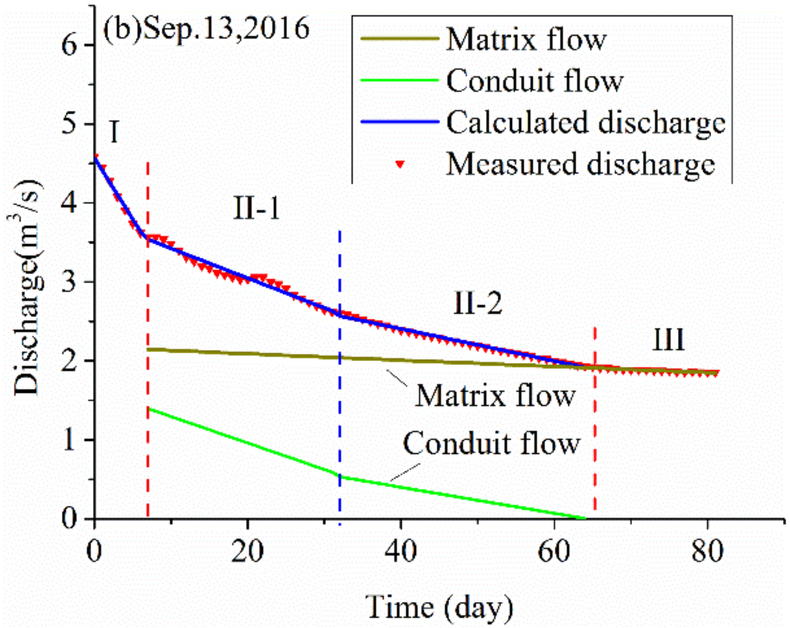
Mangin model (1975)



Fiorillo model (2011)



Our model

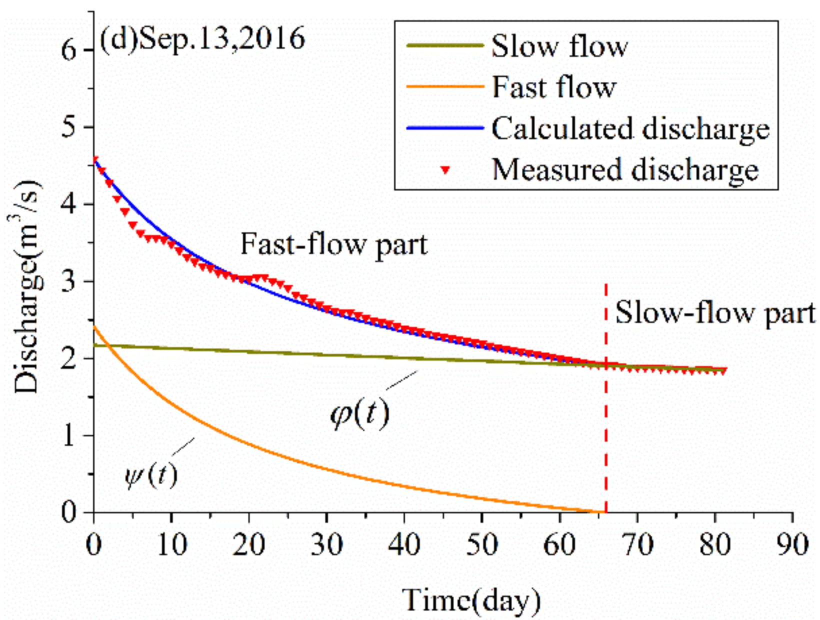


Why is our model better?

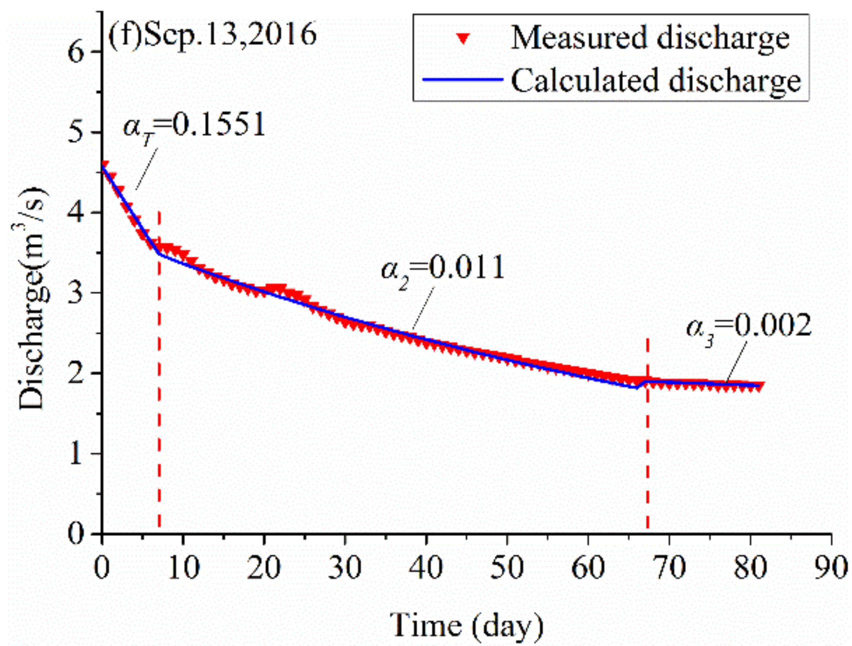
Mangin model: not considering three stages of spring discharge.

Fiorillo model: not separating conduit flow and matrix flow in the mixed-flow stage.

Mangin model (1975)

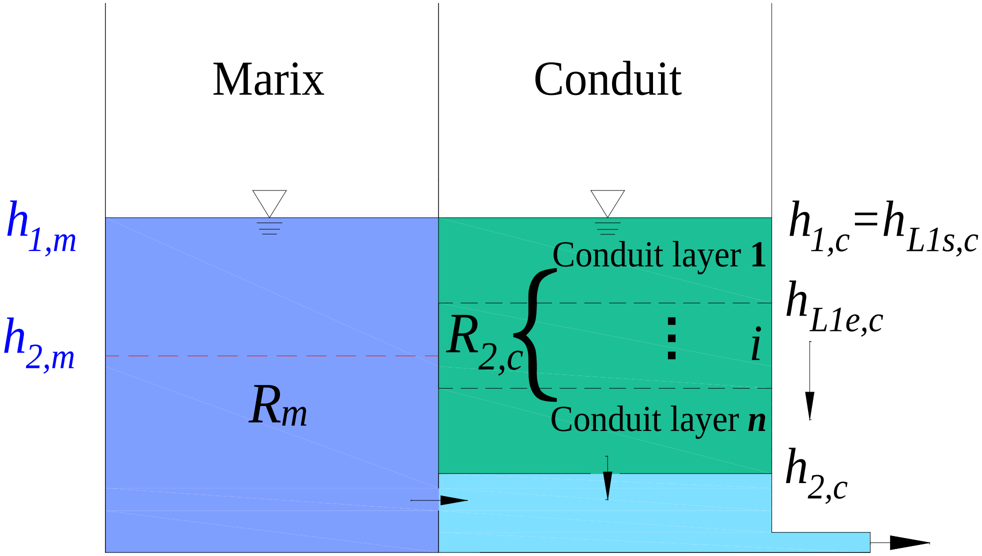


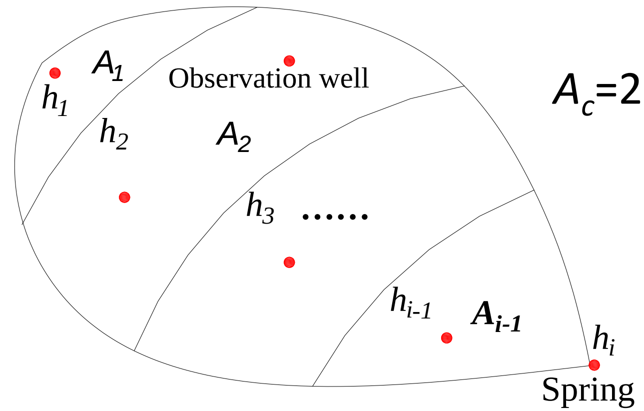
Fiorillo model (2011)



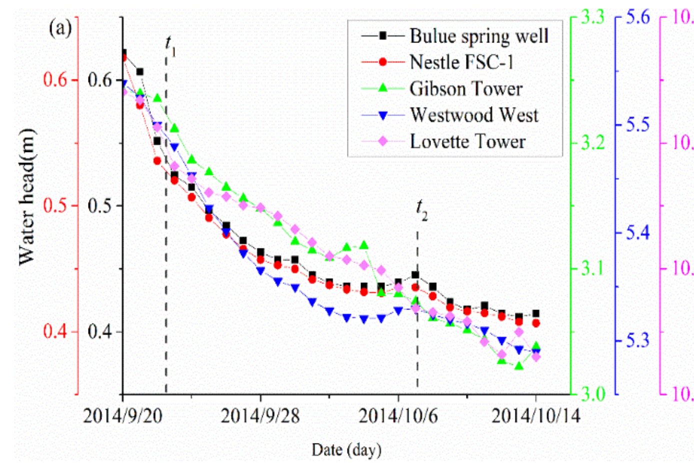
Effective Porosity of Matrix and Conduit

(b) Stage II: $t_1 \le t \le t_2$





A_c=259km^2

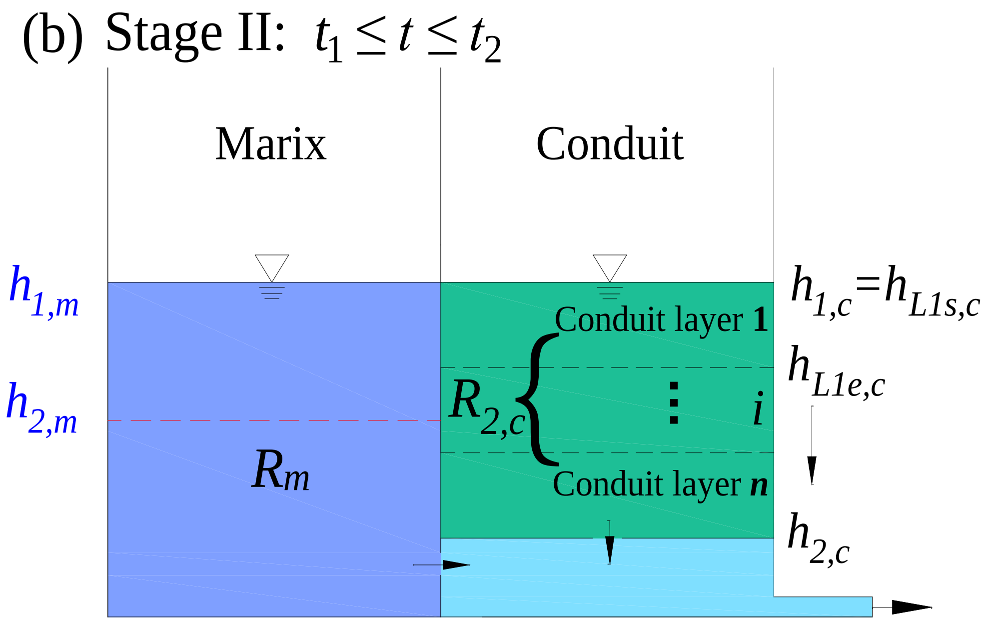


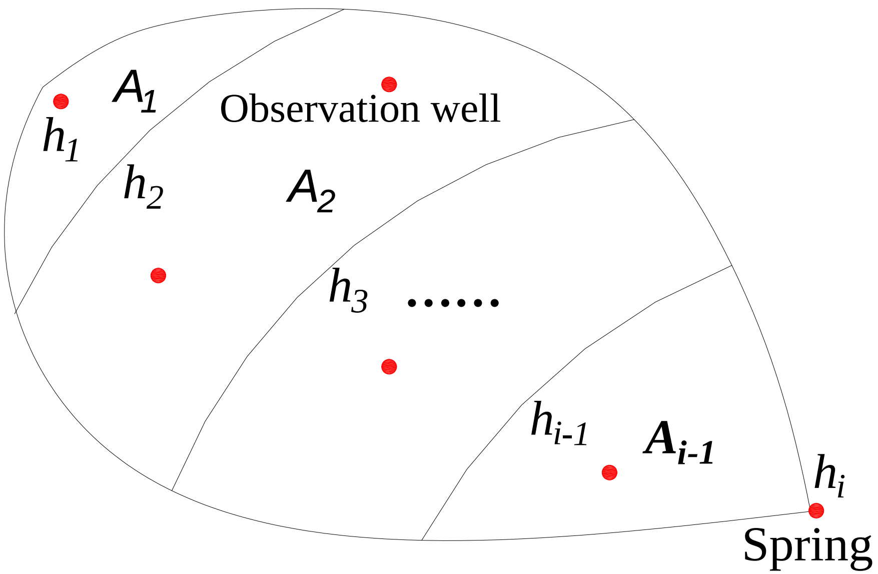
Matrix

$$n_m = \frac{V_m^{II}}{\left(h_{1,m} - h_{2,m}\right) A_c}$$

$$V_m^{II} = \int_{t_1}^{t_2} Q_{t,m}^{II} dt = \int_{t_1}^{t_2} Q_{1,\mathrm{m}} e^{-\alpha_1 (t-t_1)} dt = \frac{Q_{1,m} - Q_{2,m}}{\alpha_1}$$

Effective Porosity of Matrix and Conduit





Conduit Layer *Li*

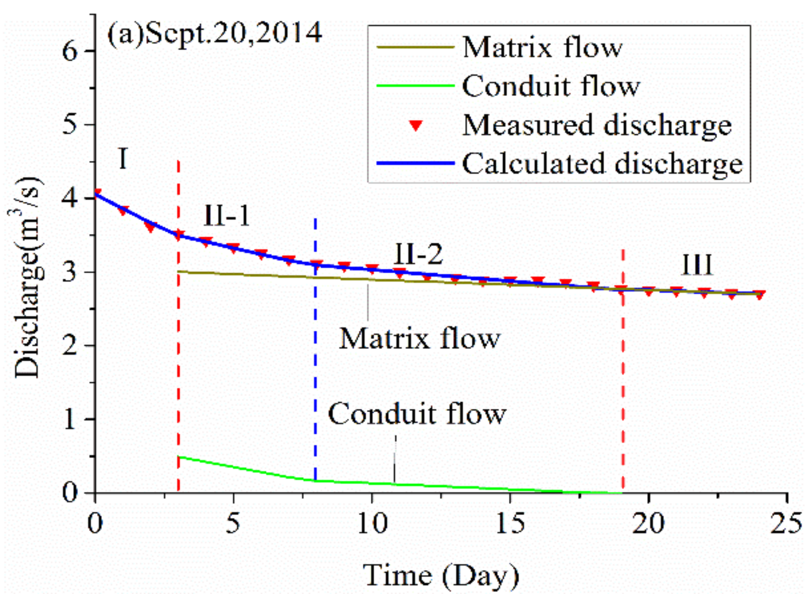
$$n_{Li,c} = \frac{V_{Li,c}^{II}}{\left(h_{Lis,c} - h_{Lie,c}\right) A_c}$$

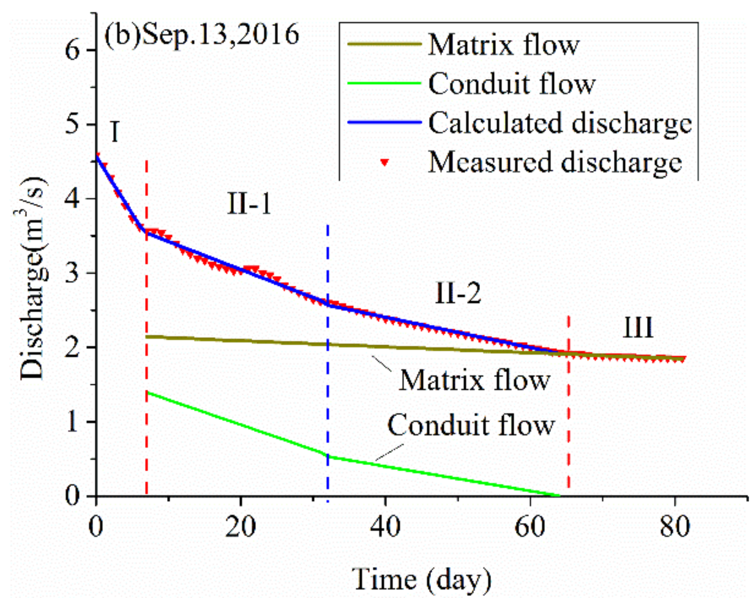
$$V_{Li,c}^{II} = \int_{t_{Lis,c}}^{t_{Lie,c}} Q_{t,Li,c}^{II} dt = \frac{Q_{Lis,c}^2 - Q_{Lie,c}^2}{2\beta_i}$$

$$h_{Lie,c} = \left(h_{Lis,c} - h_{2,c}\right)\left(\frac{Q_{t,Li,c}^{II}}{Q_{t,Li+1,c}^{II}}\right)^2 + \boxed{h_{2,c}}$$

Results

Starting Date	Head range (m)		Released water volume (m^3)		Effective porosity (%)		
	Matrix	Conduits	Matrix	Conduit	Matrix	Conduit	Total
2014/9/20	6.06-5.93	6.06-2.02	4,009,181	127,227	**11.91**	0.012	11.94
		2.02-1.00		117,461		0.048	
2016/9/13	5.72-5.36	5.72-2.86	10,306,007	2,103,438	**11.37**	0.20	11.69
		2.86-1.00		779,594		0.44	





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- Two boreholes, W-15537 and W-15515, were completed in 1984.
  - W-15537: 5% ~ 16%
  - W-15515: 1% ~ 26 %
- How do I know that the estimated conduit porosity is not wrong?
- Is it reasonable that the matrix flow is significantly larger than the conduit flow during the **mixed-flow stage**?

Conclusions

- A new model is developed for simulating the recession periods of karst spring hydrograph.
- The application of the new model to the data of the Madison Blue Spring indicates that the new model outperforms the Mangin model and Fiorillo model.
- The new model enables the estimation of effective porosity of matrix and conduit during the mixed-flow stage.
- Limitations:
  - ➢ The karst spring hydrograph must have the matrix-flow-dominated stage so that the conduit flow and matrix flow in the mixed-flow stage can be separated.
  - ➢ The model requires several parameters that cannot be directly measured, such as the area of springshed and the hydraulic head of the conduit reservoir at the beginning of the matrix-flow-dominated stage.